PHYSICAL REVIEW E VOLUME 60, NUMBER 5 NOVEMBER 1999

Two-color multistep cascading and parametric soliton-induced waveguides

Yuri S. Kivshar,[1] Andrey A. Sukhorukov,[1] and Solomon M. Saltiel[2]
[1]Australian Photonics Cooperative Research Centre, Research School of Physical Sciences and Engineering, Optical Sciences Center, Australian National University, Canberra ACT 0200, Australia
[2]Quantum Electronics Department, Faculty of Physics, University of Sofia, Sofia 1164, Bulgaria

(Received 10 May 1999)

We introduce the concept of *two-color multistep cascading* for vectorial parametric wave mixing in optical media with quadratic (second-order or $\chi^{(2)}$) nonlinear response. We demonstrate that the multistep cascading allows light-guiding-light effects with quadratic spatial solitons. With the help of the so-called ''almost exact'' analytical solutions, we describe the properties of parametric waveguides created by two-wave quadratic solitons. [S1063-651X(99)50511-4]

PACS number(s): 42.65.Tg, 05.45.Yv, 42.65.Jx, 42.65.Ky

Recent progress in the study of cascading effects in optical materials with quadratic (second-order or $\chi^{(2)}$) nonlinear response has offered new opportunities for all-optical processing, optical communications, and optical solitons [1,2]. Most of the studies of cascading effects employ parametric wave mixing processes with a single phase-matching and, as a result, two-step cascading. For example, the two-step cascading associated with type I second-harmonic generation (SHG) includes the generation of the second harmonic ($\omega+\omega=2\omega$) followed by reconstruction of the fundamental wave through the down-conversion frequency mixing (DFM) process ($2\omega-\omega=\omega$). These two processes are governed by one phase-matched interaction and they differ only in the direction of power conversion.

The idea to explore more than one simultaneous nearly phase-matched process, or *double-phase-matched (DPM) wave interaction*, became attractive only recently [3,4], for the purposes of all-optical transistors, enhanced nonlinearity-induced phase shifts, and polarization switching. In particular, it was shown [4] that multistep cascading can be achieved by two second-order nonlinear cascading processes, SHG and sum-frequency mixing (SFM), and these two processes can also support a novel class of multicolor parametric solitons [5]. The physics involved into the multistep cascading can be understood by analyzing a chain of parametric processes: SHG ($\omega+\omega=2\omega$)$\rightarrow$SFM ($\omega+2\omega=3\omega$)$\rightarrow$DFM ($3\omega-\omega=2\omega$)$\rightarrow$DFM ($2\omega-\omega=\omega$). The main disadvantage of this kind of parametric processes for applications is that it requires nonlinear media transparent up to the third harmonic frequency.

Then, the important question is: *Can we find parametric processes which involve only two frequencies but allow us to get all advantages of multistep cascading?* In this Rapid Communication, we answer positively this question introducing the concept of *two-color multistep cascading*. We demonstrate a number of unique features of multistep parametric wave mixing that do not exist for the conventional two-step cascading. In particular, using one of the processes of two-color multistep cascading, we show how to introduce and explore the concept of light guiding light for quadratic spatial solitons, which was analyzed earlier for Kerr-like spatial solitary waves [6] but seemed impossible for parametric interactions. We find ''almost exact'' analytical solutions for two-wave quadratic solitons and investigate, analytically and numerically, the properties of parametric waveguides created by quadratic spatial solitons in $\chi^{(2)}$ nonlinear media.

To introduce more than one parametric process involving only two frequencies, we consider a vectorial interaction of waves with different polarization. We denote two orthogonal polarization components of the fundamental frequency (FF) wave ($\omega_1=\omega$) as A and B, and two orthogonal polarizations of the second harmonic (SH) wave ($\omega_2=2\omega$), as S and T. Then, a simple multistep cascading process consists of the following steps. First, the FF wave A generates the SH wave S via type I SHG process. Then, by down-conversion SA-B, the orthogonal FF wave B is generated. At last, the initial FF wave A is reconstructed by the processes SB-A or AB-S, SA-A. Two principal second-order processes AA-S and AB-S correspond to *two different components* of the $\chi^{(2)}$ susceptibility tensor, thus introducing additional degrees of freedom into the parametric interaction. Different types of multistep cascading processes are summarized in Table I. The processes in row (a) of Table I described above and the multistep cascading introduced in Ref. [4] are qualitatively similar, but the latter involves a third-harmonic wave.

To demonstrate some of the unique properties of the multistep cascading, we discuss here how it can be employed for light-guiding-light effects in quadratic media. For this purpose, we consider the principal DPM process (c) (see Table I) in the planar slab-waveguide geometry. Using the slowly varying envelope approximation with the assumption of zero absorption of all interacting waves, we obtain

$$2ik_1\frac{\partial A}{\partial z}+\frac{\partial^2 A}{\partial x^2}+\chi_1 SA^* e^{-i\Delta k_1 z}=0, \quad (1)$$

TABLE I. Possible multistep cascading processes.

	Principal	Equivalent
(a)	(AA-S, AB-S)	(BB-S, AB-S); (AA-T, AB-T) (BB-T, AB-T)
(b)	(AA-S, AB-T)	(BB-S, AB-T); (AA-T, AB-S) (BB-T, AB-S)
(c)	(AA-S, BB-S)	(AA-T, BB-T)
(d)	(AA-S, AA-T)	(BB-S, BB-T)

$$2ik_1\frac{\partial B}{\partial z}+\frac{\partial^2 B}{\partial x^2}+\chi_2 SB^* e^{-i\Delta k_2 z}=0,$$

$$4ik_1\frac{\partial S}{\partial z}+\frac{\partial^2 S}{\partial x^2}+2\chi_1 A^2 e^{i\Delta k_1 z}+2\chi_2 B^2 e^{i\Delta k_2 z}=0,$$

where $\chi_{1,2}=2k_1\sigma_{1,2}$, the nonlinear coupling coefficients σ_k are proportional to the elements of the second-order susceptibility tensor, and Δk_1 and Δk_2 are the corresponding wave-vector mismatch parameters.

To simplify system (1), we look for its stationary solutions and introduce the normalized envelopes u, v, and w according to the following relations: $A=\gamma_1 u\exp(i\beta z-i/2\Delta k_1 z)$, $B=\gamma_2 v\exp(i\beta z-i/2\Delta k_2 z)$, and $S=\gamma_3 w\exp(2i\beta z)$, where $\gamma_1^{-1}=2\chi_1 x_0^2$, $\gamma_2^{-1}=2x_0^2(\chi_1\chi_2)^{1/2}$, and $\gamma_3^{-1}=\chi_1 x_0^2$, and the longitudinal and transverse coordinates are measured in the units of $z_0=(\beta-\Delta k_1/2)^{-1}$ and $x_0=(z_0/2k_1)^{1/2}$, respectively. Then, we obtain a system of normalized equations,

$$i\frac{\partial u}{\partial z}+\frac{\partial^2 u}{\partial x^2}-u+u^* w=0,$$

$$i\frac{\partial v}{\partial z}+\frac{\partial^2 v}{\partial x^2}-\alpha_1 v+\chi v^* w=0, \qquad (2)$$

$$2i\frac{\partial w}{\partial z}+\frac{\partial^2 w}{\partial x^2}-\alpha w+\frac{1}{2}(u^2+v^2)=0,$$

where $\chi\equiv(\chi_2/\chi_1)$, $\alpha_1=(\beta-\Delta k_2/2)(\beta-\Delta k_1/2)^{-1}$, and $\alpha=4\beta(\beta-\Delta k_1/2)^{-1}$. Equations (2) are the fundamental model for describing any type of multistep cascading processes of type (c) (see Table I).

First of all, we notice that for $v=0$ (or, similarly, $u=0$), the dimensionless model (2) coincides with the corresponding model for the two-step cascading due to type I SHG discussed earlier [1,2], and its stationary solutions are defined by the equations for real u and w,

$$\frac{d^2u}{dx^2}-u+uw=0,$$
$$\frac{d^2w}{dx^2}-\alpha w+\frac{1}{2}u^2=0, \qquad (3)$$

which possess a one-parameter family of two-wave localized solutions (u_0,w_0) found earlier numerically for any $\alpha\neq 1$, and also known analytically for $\alpha=1$, $u_0(x)=(3/\sqrt{2})\mathrm{sech}^2(x/2)=\sqrt{2}w_0(x)$ (see Ref. [2]).

Then, in the small-amplitude approximation, the equation for real orthogonally polarized FF wave v can be treated as an eigenvalue problem for an effective waveguide created by the SH field $w_0(x)$,

$$\frac{d^2v}{dx^2}+[\chi w_0(x)-\alpha_1]v=0. \qquad (4)$$

Therefore, an additional parametric process allows us to propagate a probe beam of one polarization in *an effective waveguide* created by a two-wave spatial soliton in a quadratic medium with FF component of another polarization. However, this type of waveguide is different from what has been studied for Kerr-like solitons because it is *coupled parametrically* to the guided modes and, as a result, the physical picture of the guided modes is valid, rigorously speaking, only in the case of stationary phase-matched beams. As a result, the stability of the corresponding waveguide and localized modes of the orthogonal polarization it guides is a key issue. In particular, the waveguide itself (i.e., two-wave parametric soliton) becomes unstable for $\alpha<\alpha_{\mathrm{cr}}\approx 0.2$ [7].

In order to find the guided modes of the parametric waveguide created by a two-wave quadratic soliton, we have to solve Eq. (4) where the solution $w_0(x)$ is known numerically only. These solutions have been also described by the variational method [8], but the different types of the variational ansatz used do not provide a very good approximation for the soliton profile at all α. For our eigenvalue problem (4), the function $w_0(x)$ defines parameters of the guided modes and, in order to obtain accurate results, it should be calculated as close as possible to the exact solutions found numerically. To resolve this difficulty, below we suggest an ''almost exact'' solution that *would allow us to solve analytically many of the problems involving quadratic solitons*, including the eigenvalue problem (4).

First, we notice that from the exact result at $\alpha=1$ and the asymptotic result for large α, $w\approx u^2/(2\alpha)$, it follows that the SH component $w_0(x)$ of Eqs. (3) remains almost self-similar for $\alpha\geqslant 1$. Thus, we look for the SH field in the form $w_0(x)=w_m\,\mathrm{sech}^2(x/p)$, where w_m and p are unknown yet parameters. The solution for $u_0(x)$ should be consistent with this choice of the shape for SH, and it is defined by the first (linear for u) equation of the system (3). Therefore, we can take u in the form of the lowest guided mode, $u_0(x)=u_m\,\mathrm{sech}^p(x/p)$, which corresponds to an effective waveguide $w_0(x)$. By matching the asymptotics of these trial functions with those defined directly from Eqs. (3) at small and large x, we obtain the following solution:

$$u_0(x)=u_m\,\mathrm{sech}^p(x/p), \quad w_0(x)=w_m\,\mathrm{sech}^2(x/p), \qquad (5)$$

$$u_m^2=\frac{\alpha w_m^2}{(w_m-1)}, \quad \alpha=\frac{4(w_m-1)^3}{(2-w_m)}, \quad p=\frac{1}{(w_m-1)}, \qquad (6)$$

where all parameters are functions of α only. It it easy to verify that, for $\alpha_{\mathrm{cr}}<\alpha<\infty$, the SH amplitude varies in the region $1.3<w_m<2$, so that all the terms in Eq. (6) remain positive.

It is really amazing that the analytical solution (5) and (6) provides *an excellent approximation* for the profiles of the two-wave parametric solitons found numerically. Figures 1(a) and 1(b) show a comparison between the maximum amplitudes of the FF and SH components and selected soliton profiles, respectively. As a matter of fact, the numerical and analytical results on these plots are not distinguishable, and that is why we show them differently, by continuous curves and crosses. For $\alpha<1$, the SH profile changes, but in the region $\alpha>\alpha_{\mathrm{cr}}$ the approximate analytical solution is still

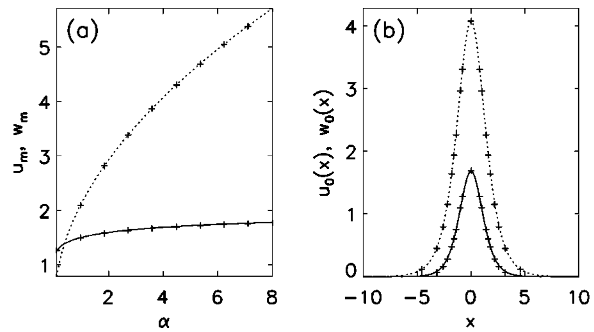

FIG. 1. Comparison between the numerical (continuous curves) and ''almost exact'' analytical (crosses) solutions for two-wave (FF, dotted; SH, solid) parametric solitons: (a) maximum amplitudes, (b) two-wave soliton profile at $\alpha=4$.

very close to the exact numerical one: a relative error is less than 1%, for the amplitudes, and it does not exceed 3%, for the power components. That is why we define the analytical solution given by Eqs. (5) and (6) as ''almost exact.'' Details of the derivation, as well as the analysis of the case $\alpha<1$, will be presented elsewhere [9].

Now, the eigenvalue problem (4) can be readily solved analytically. The eigenmode cutoff values are defined by the parameter α_1 that takes one of the discrete values, $\alpha_1^{(n)}=(s-n)^2/p^2$, where $s=-(1/2)+[(1/4)+w_m\chi p^2]^{1/2}$. Number n stands for the mode order ($n=0,1,\dots$), and the localized solutions are possible provided $n<s$. The profiles of the guided modes can be found analytically in the form

$$v_n(x)=V\mathrm{sech}^{s-n}(x/p)H(-n,2s-n+1,s-n+1;\zeta),$$

where $\zeta=\frac{1}{2}[1-\tanh(x/p)]$, V is the mode amplitude, and H is the hypergeometric function.

According to these results, a two-wave parametric soliton creates, in general, a multimode waveguide and larger number of the guided modes is observed for smaller α. Figures 2(a) and 2(b) show the dependence of the mode cutoff values $\alpha_1^{(n)}$ versus α, at fixed χ, and versus the parameter χ, at fixed α, respectively. For the case $\chi=1$, the dependence has a simple form: $\alpha_1^{(n)}=[1-n(w_m-1)]^2$.

Because a two-wave soliton creates an induced waveguide parametrically coupled to the modes of the orthogonal polarization it guides, the dynamics of the guided modes *may differ drastically* from that of conventional waveguides based on the Kerr-type nonlinearities. Figures 3(a)–3(d)

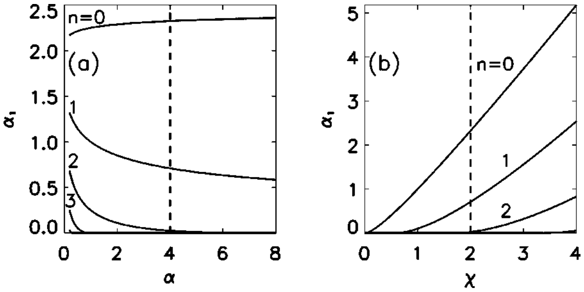

FIG. 2. Cutoff eigenvalues $\alpha_1^{(n)}$ of the guided modes shown as (a) functions of α at $\chi=2$, and (b) functions of χ at $\alpha=4$. Dashed lines correspond to the intersection of the plots in the parameter space (α,χ).

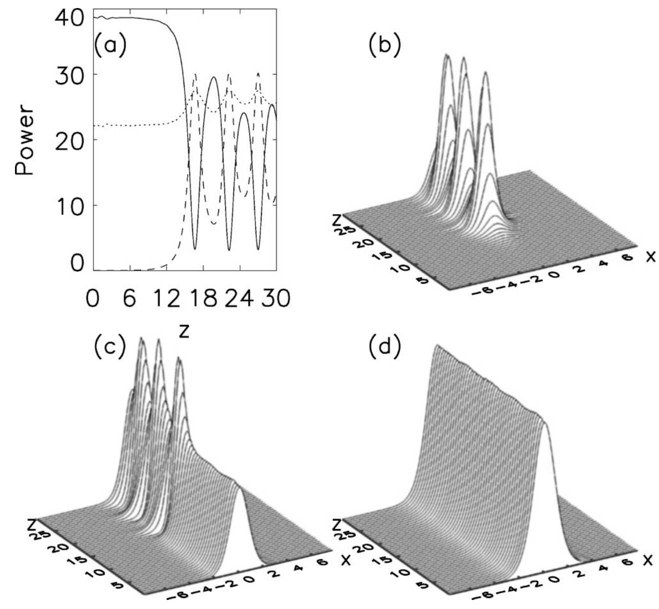

FIG. 3. (a) Change of the normalized power in FF (u, solid lines) and SH (w, dotted lines) components, which initially constitute a two-wave soliton, and in the guided mode (v, dashed lines) at $\chi=2$, demonstrating amplification of a guided wave. Evolution of the guided wave and effective waveguide (SH) is presented in plots (b) and (c), respectively. (d) Stationary propagation of a stable fundamental mode $(\chi=1)$. For all the plots $\alpha=4$, the initial amplitude is $v_0=0.1$, and α_1 corresponds to the bifurcation point.

show two examples of the evolution of guided modes. In the first example [see Fig. 3(a)–3(c)], a weak fundamental mode is amplified via parametric interaction with a soliton waveguide, and the mode experiences a strong power exchange with the orthogonally polarized FF component through the SH field, but with only a weak deformation of the induced waveguide [see Figs. 3(a), dotted curve]. This effect can be interpreted as a power exchange between two guided modes of orthogonal polarizations in a waveguide created by the SH field. In the second example, the propagation is stable [see Fig. 3(d)].

When all the fields in Eq. (2) are not small, i.e., the small-amplitude approximation is no longer valid, the profiles of the three-component solitons should be found numerically. However, some of the lowest-order states can be calculated approximately using the approach of the ''almost exact'' solution (5) and (6) described above. Moreover, a number of the solutions and their families can be obtained in *an explicit analytical form*. For example, for $\alpha_1=1/4$, there exist two *families of three-component solitary waves* for any $\alpha\geqslant1$, which describe soliton branches starting at the bifurcation points $\alpha_1=\alpha_1^{(1)}$ at $\alpha=1$: (i) the soliton with a zero-order guided mode for $\chi=1/3$: $u(x)=(3/\sqrt{2})\mathrm{sech}^2(x/2)$, $v(x)=c_2\,\mathrm{sech}(x/2)$, $w(x)=(3/2)\,\mathrm{sech}^2(x/2)$; and (ii) the soliton with a first-order guided mode for $\chi=1$: $u(x)=c_1\,\mathrm{sech}^2(x/2)$, $v(x)=c_2\mathrm{sech}^2(x/2)\sinh(x/2)$, $w(x)=(3/2)\,\mathrm{sech}^2(x/2)$, where $c_2=\sqrt{3(\alpha-1)}$ and $c_1=\sqrt{(9/2)+c_2^2}$. Some other soliton solutions exist for a specific choice of the parameters, e.g., for $\alpha=\alpha_1=4/9$ and $\chi=1$, we find $u(x)=(4/3)\mathrm{sech}^3(x/3)$, $v(x)=(4/3)\mathrm{sech}^3(x/3)\sinh(x/3)$, and $w(x)=(4/3)\mathrm{sech}^2(x/3)$. Stability of these three-wave solitons is a nontrivial issue; a rigorous analysis of all such multicomponent states is beyond the scope of the present Rapid Communication and will be addressed elsewhere.

At last, we would like to mention that in the limit of large α, when the coupling to the second harmonic is weak, we can use the cascading approximation $w \approx (u^2+v^2)/(2\alpha)$. Then, the equations for two orthogonal polarizations of the FF wave reduce to a system of two coupled NLS equations, an asymmetric case of TE-TM vector spatial solitons well studied in the literature (see, e.g., Ref. [10] and references therein).

For a practical realization of the DPM processes and the soliton light-guiding-light effects described above, we can suggest two general methods. The first method is based on the use of *two commensurable periods* of the quasi-phase-matched (QPM) periodic grating. Indeed, to achieve DPM, we can employ the first-order QPM for one parametric process, and the third-order QPM, for the other parametric process. Taking, as an example, the parameters for $LiNbO_3$ and AA-S $(xx-z)$ and BB-S $(zz-z)$ processes [11], we find two points for DPM at about 0.89 and 1.25 μm. This means that a single QPM grating can provide simultaneous phase-matching for two parametric processes. For such a configuration, we obtain $\chi \approx 1.92$ or, interchanging the polarization components, $\chi \approx 0.52$. The second method to achieve the conditions of DPM processes is based on the idea of *quasi-periodic QPM grating*. As has been recently shown experimentally [12] and numerically [13], Fibonacci optical superlattices provide an effective way to achieve phase-matching at *several incommensurable periods* allowing multifrequency harmonic generation in a single structure.

In conclusion, we have introduced the concept of two-color multistep cascading and demonstrated a possibility of light-guiding-light effects with parametric waveguides created by two-wave spatial solitons in quadratic media. We believe our results open a new direction in research of cascading effects, and may bring new ideas into other fields of nonlinear physics, where parametric wave interactions are important.

[1] For a comprehensive overview of cascading, see G. Stegeman, D.J. Hagan, and L. Torner, Opt. Quantum Electron. **28**, 1691 (1996).

[2] For an overview of quadratic spatial solitons, see L. Torner, in *Beam Shaping and Control with Nonlinear Optics*, edited by F. Kajzer and R. Reinisch (Plenum, New York, 1998), p. 229; Yu.S. Kivshar, in *Advanced Photonics with Second-Order Optically Nonlinear Processes*, edited by A.D. Boardman, L. Pavlov, and S. Tanev (Kluwer, Dordrecht, 1998), p. 451.

[3] See, e.g, G. Assanto, I. Torelli, and S. Trillo, Opt. Lett. **19**, 1720 (1994); A.D. Boardman, P. Bontemps, and K. Xie, Opt. Quantum Electron. **30**, 891 (1998).

[4] K. Koynov and S. Saltiel, Opt. Commun. **152**, 96 (1998).

[5] Yu.S. Kivshar, T. Alexander, and S. Saltiel, Opt. Lett. **24**, 759 (1999).

[6] For an overview, see M. Segev and G. Stegeman, Phys. Today **51**(8), 42 (1998); A.W. Snyder and F. Ladouceur, Opt. Photonics News **10** (2), 35 (1999).

[7] D.E. Pelinovsky, A.V. Buryak, and Yu.S. Kivshar, Phys. Rev. Lett. **75**, 591 (1995).

[8] V.V. Steblina *et al.*, Opt. Commun. **118**, 345 (1995); A.D. Boardman *et al.*, Phys. Rev. A **52**, 4099 (1995); V.M. Agranovich *et al.*, Phys. Rev. B **53**, 15 451 (1996).

[9] A.A. Sukhorukov (unpublished).

[10] N.N. Akhmediev, Opt. Quantum Electron. **30**, 535 (1998).

[11] V.G. Dmitriev, G.G. Gurzadyan, and D.N. Nikogosyan, *Handbook of Nonlinear Optical Crystals* (Springer, Berlin, 1997).

[12] S. Zhu *et al.*, Phys. Rev. Lett. **78**, 2752 (1997); Science **278**, 843 (1997); Y. Zhu *et al.*, Appl. Phys. Lett. **73**, 432 (1998).

[13] C. Balslev Clausen, Yu.S. Kivshar, O. Bang, and P.L. Christiansen (unpublished).